MTH 520/622 - INTRODUCTION TO HYPERBOLIC GEOMETRY

SEMESTER 1, 2017-18

August 18, 2017

General information

Classroom: AB1 - 308
Schedule: Tue: 5:00-5:55 PM, Wed: 12:00-1:00 PM & Fri: 11:00-11:55 AM
Webpage: `http://home.iiserb.ac.in/~kashyap/MTH 520/mth520.html`

Contact information:

Instructor: Dr. Kashyap Rajeevsarathy
Office: Academic Building 1, Room 314
Office hours: Mon, 10:00 - 11:00 AM
E-mail: `kashyap@iiserb.ac.in`

Topics

- **The general Möbius group.** The extended complex plane (or the Riemman sphere) $\hat{\mathbb{C}}$; The general Möbius group $\text{Mob}^+(\hat{\mathbb{C}})$; Identifying $\text{Mob}^+(\hat{\mathbb{C}})$ with the matrix group $\text{PGL}(2,\mathbb{C})$; Classification of elements of elements of $\text{Mob}^+(\hat{\mathbb{C}})$; Reflections and the general Möbius group $\text{Mob}(\hat{\mathbb{C}})$; Conformality of elements in $\text{Mob}(\hat{\mathbb{C}})$.

- **The upper-half plane model $\mathbb{H}^2$.** The upper half plane $\mathbb{H}^2$; The subgroup $\text{Mob}(\mathbb{H}^2)$; Transitivity properties of $\text{Mob}^+(\mathbb{H}^2)$; Geometry of the action of $\text{Mob}^+(\mathbb{H}^2)$; The metric in $\mathbb{H}^2$; Element of arc-length in $\mathbb{H}^2$; Path metric in $\mathbb{H}^2$; The Poincaré metric $d_{\mathbb{H}}$ on $\mathbb{H}^2$; Geodesics in $\mathbb{H}^2$;

Identifying the group $\mathrm{Mob}(\mathbb{H}^2)$ of isometries of $(\mathbb{H}^2, d_{\mathbb{H}})$ with $\mathrm{PSL}(2;\mathbb{R})$; Ultraparallel lines in $\mathbb{H}^2$.

- **The Poincaré disk model $\mathbb{D}$.** The Poincaré disk $\mathbb{D}$; Transitioning from $\mathbb{H}^2$ to $\mathbb{D}$ via $\mathrm{Mob}^+(\mathbb{H}^2)$; Element of arc-length and the metric $d_{\mathbb{D}}$ in $\mathbb{D}$; Group $\mathrm{Mob}(\mathbb{D})$ of isometries of $(\mathbb{D}, d_{\mathbb{D}})$; Geodesics in $\mathbb{D}$; Centre, radii, and length of hyperbolic circles in $\mathbb{D}$; Hyperbolic structures on holomorphic disks.

- **Properties of $\mathbb{H}^2$.** Curvature of $\mathbb{H}^2$; Convex subsets of $\mathbb{H}^2$; Hyperbolic polygons; Area of a subset of $\mathbb{H}^2$; Gauss-Bonnet formula - area of a hyperbolic triangle; Applications of Gauss-Bonnet Formula: Area of reasonable hyperbolic polygons, existence of certain hyperbolic n-gons, hyperbolic dilations; Putting a hyperbolic structure on a surface using hyperbolic polygons; Hyperbolic trigonometry: triogometric identities, law of sines and cosines, Pythagorean theorem.

Suggested books

1. James W. Anderson, *Hyperbolic Geometry (2^{nd} Edition)*, Springer, 2005.
2. Arlan Ramsay, Robert D. Richtmyer, *Introduction to Hyperbolic Geometry*, Springer, 1995.
3. Harold E. Wolfe, *Introduction to Non-Euclidean Geometry*, Dover, 2012.
4. Alan F. Beardon, *The geometry of discrete groups (Chapter 7)*, Springer, 1983.
5. Svetlana Katok, *Fuchsian Groups (Chapter 1)*, Chicago Lectures in Mathematics, 1992.
6. John Stillwell, *Geometry of surfaces (Chapter 4)*, Springer, 1992.

Homework policy

- Homework assignments will be due every other week. The problems to be turned in and the due dates will be posted on the course webpage. So it is your responsibility to regularly check the course webpage for any updates.

- If you must miss the class on a due date, try turning in your assignment in advance or have some one else turn it in for you.
- Problems written should be legible and must clearly indicate the steps used to arrive at the solution.

Quiz and exam policy

- Up to two quizzes may be administered during the course of the semester - one before the midterm and another before the final. The syllabus for the quizzes with be announced in class.
- The schedule for the midterm and final exams will be as per the academic calendar.
- The topics for the midterm exam will be announced in class in due course. However, the final exam will be comprehensive with more emphasis on topics that will be discussed after the midterm exam.
- No books, notes, or electronic devices of any kind may be used during exams.
- When graded exams or quizzes are returned, please check them carefully for any grading errors. All grading issues should be brought to my attention as soon as possible. Note that your scores are not renegotiable after the final grades are submitted.
- Do not make travel plans that might prevent you from taking any scheduled exam or quiz. If you have a verifiable reason why you cannot be present at an exam, you must contact me in advance to make an alternative arrangement.

Grading Scheme:

A total of 100 percentage points will be distributed as follows:

- Homework - 20 %
- Midterm - 30 %
- Final Exam - 50 %